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general rate of convergence of particular classes of terms, will give $+0''\cdot02 \pm 0''\cdot02$. Thus the coefficient is $-127''\cdot45 \pm 0''\cdot02$. Multiplying this by $(E-M)/(E+M) = 80\cdot15/82\cdot15$, and reducing to the parallax $8''\cdot790$, I find

$$\text{parallactic inequality} \doteq -124''\cdot92 \, \frac{\text{solar parallax}}{8''\cdot790} \sin D$$

with a possible error of $0''\cdot02$.

In the *Monthly Notices* for 1903 December Mr. Cowell finds $-124''\cdot75$ for the observed value of this coefficient. Comparing we obtain

$$\text{solar parallax} = 8''\cdot778.$$

Haverford College: 1904 *April* 1.

Some further Analyses of the Moon's Errors of Longitude, 1847–1901. By P. H. Cowell.

The columns 2–7 of the annexed table gives the "apparent" coefficient of $\sin 2\theta$, $\cos 2\theta$, $\sin(4D-2g)$, $\cos(4D-2g)$, $\sin D$ and the apparent error of semi-diameter as deduced from each of the forty-eight periods of analysis, consisting of 400 lunar days each, into which the period 1847–1901 has been divided.

Last month I explained how an apparent coefficient of $\sin\phi$ might in reality be due to a term $\sin(\phi \pm D)$. Periodicity in the apparent coefficient of $\sin\phi$ indicates a term different slightly in argument from ϕ or from $\phi \pm D$.

The angle θ is an auxiliary angle whose movement is $13^\circ\cdot5$ in a lunar day. θ therefore recurs in eighty days and 2θ in forty days; θ differs very slightly in movement from g, the mean anomaly, for the movement of g in 400 lunar days $= 13\cdot5 \times 400 + 9^\circ\cdot1681$. $4D-2g$ is double the evection.

The first four coefficients tabulated appear to be nearly, if not entirely, accidental. I cannot find any marked periodicity in any one of the four series. The conclusion from this is that the tables of Hansen contain either no error (with coefficient exceeding $0''\cdot3$ say) and argument $2g$, $4D-2g$, $2g \pm D$, $4D-2g \pm D$ or angles differing from any of the above by an argument of long period, such as $\omega-\omega'$ or 2ω, or that if an error in one term does exist, its apparent effect is cancelled (within a limit of about $0''\cdot3$) by another error in an allied term or errors in both the allied terms—an improbable supposition. Hansen's tables are therefore probably correct as regards terms of the above form. Subsequently I intend to give analyses of the allied terms $g+g'$ and $g-3g'$.

Apart from accidental error, therefore, I infer that all these

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In Tenths of a Second of Arc.

Period.	Apparent Coefficients of					Apparent Corr. to Semi-diameter.	Resolved Coefficient of sin D.	Resolved Corr. to Semi-diameter.
	sin 2θ.	cos 2θ.	sin $(4D-2g)$.	(cos $4D-2g$).	sin D.			
86	−2	+3	+2	−2	+ 2	+ 1	+ 5	− 2
87	−4	+6	−7	−9	−11	− 8	− 7	− 3
88	0	+2	0	+2	− 1	− 2	+ 8	− 8
89	+4	+3	0	−4	0	0	0	0
90	−6	−2	−2	+3	− 5	− 4	− 1	− 4
91	+5	−1	+3	+3	+ 5	+ 3	+ 8	− 2
92	+4	−5	−4	−1	− 8	− 5	−10	+ 2
93	−2	−1	0	+1	0	− 2	+14	−12
94	+3	−1	+5	−2	− 5	− 3	− 8	+ 2
95	+8	−8	0	0	− 9	− 6	− 9	0
96	−3	−1	−1	−2	−12	− 8	−12	0
97	−9	+2	+1	+2	−13	−11	+ 3	−14
98	+3	+4	+5	+3	− 9	− 6	− 9	0
99	−3	−3	+3	0	− 1	0	− 6	+ 4
100	−1	+5	−2	−1	+ 4	+ 3	+ 2	+ 2
101	+8	−4	+2	0	− 1	0	− 6	+ 4
102	−5	+1	−2	−3	+ 4	+ 3	+ 2	+ 2
103	−6	0	−1	0	+ 4	+ 3	+ 2	+ 2
104	+1	+7	0	−2	− 6	− 3	−14	+ 6
105	+9	−1	+1	−3	+ 1	+ 1	− 1	+ 2
106	−2	−3	−2	0	+ 3	+ 1	+10	− 6
107	0	+4	0	+1	+ 2	+ 1	+ 5	− 2
108	−5	−6	+2	−4	+ 6	+ 4	+ 7	0
109	+1	+4	0	0	+13	+ 8	+18	− 4
110	−3	0	−7	+3	+ 2	+ 3	− 9	+ 9
111	+5	+4	−2	−4	− 3	− 2	− 3	0
112	−1	−4	0	+8	− 1	0	− 6	+ 4
113	−7	+1	0	+2	+ 2	+ 1	+ 5	− 2
114	−1	+5	−1	−3	+ 3	+ 2	+ 3	0
115	−2	−3	+6	+3	+ 3	+ 3	− 4	+ 5
116	0	+5	0	−1	+ 3	+ 3	− 4	+ 5
117	+5	+1	−6	+1	+ 4	+ 4	− 5	+ 8
118	+1	−6	+3	+1	+ 4	+ 3	+ 2	+ 2
119	−1	−2	−6	0	+ 2	+ 1	+ 5	− 2
120	−5	+4	+4	+3	0	0	0	0
121	−4	+1	+2	−1	+ 5	+ 3	+ 8	− 2
122	−5	+1	+2	−1	+12	+ 8	+12	0
123	−5	+5	+1	0	+ 5	+ 3	+ 8	− 2
124	+5	+4	−2	−4	+ 5	+ 3	+ 8	− 2
125	−2	+1	+1	+1	+ 1	+ 1	− 1	+ 2
126	−1	+4	+3	−4	− 6	− 4	− 7	0
127	+5	+2	+4	−1	+ 2	+ 2	− 2	+ 4
128	+2	−4	−1	+3	− 1	0	− 6	+ 4
129	−5	−3	+1	+1	− 4	− 4	+ 5	− 8
130	0	+3	+2	+2	− 5	− 3	− 8	+ 2
131	+2	−2	0	−3	− 6	− 5	+ 1	− 6
132	−6	+1	0	0	− 2	− 1	− 5	+ 2
133	+7	+4	−2	−6	− 2	− 1	− 5	+ 2

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quantities should be zero. The probable error of the apparent coefficient of any periodic term can therefore be deduced. Adding the four columns without regard to sign and dividing by 48, and then multiplying by 0·85 to reduce "Mean of the Errors" to "Probable Error," I obtain from the apparent coefficient of

$\sin 2\theta$,	$174 \div 48 \times 0{\cdot}85$			$= \pm 0''{\cdot}31$
$\cos 2\theta$,	147	,,	,,	$= \pm 0''{\cdot}26$
$\sin (4D - 2g)$,	101	,,	,,	$= \pm 0''{\cdot}18$
$\cos (4D - 2g)$,	104	,,	,,	$= \pm 0''{\cdot}18$

and from the mean of these four separate determinations the probable error of any entry in the first four columns of the table seems to be about $\pm 0''{\cdot}23$.

With this last number as the basis, the probable error of a single "tabular — observed" error of the Moon appears to be about $\pm 0''{\cdot}23 \times \sqrt{\frac{1}{2}\cdot\frac{5647}{48}} = \pm 1''{\cdot}7$. In my December paper I gave $1''{\cdot}5$ as an estimate of this quantity.

Again, the probable error of a coefficient deduced from all forty-eight periods is

$$\pm 0''{\cdot}23 \div \sqrt{48} = \pm 0''{\cdot}03$$

When the Airy period 1750–1851 is fully worked up there will be plenty of material for checking this estimate. At present I can only refer to the values of the principal elliptic coefficient and the coefficient of the Jovian evection in my paper of last month, from which it will be seen that $0''{\cdot}03$ is a fair estimate of the probable error.

As regards the tables of the Moon the results of these two analyses are of a negative character only, indicating the absence and not the presence of periodic errors. It is interesting to note, however, that had Airy analysed for the argument $4D - 2g$ he would have found a periodic error of $4''$ in his tabular places, and this would probably have led to a more thorough analysis.

Coming now to the last four columns of the table the sixth column exhibits the apparent coefficient of sin D, that is to say, the quantity obtained by multiplying each error by sin D and dividing by half the number of observations. It is evident that an error of semi-diameter, or a term sin 2D, will each contribute to the "apparent coefficient of sin D." The seventh column exhibits the apparent error of semi-diameter, that is to say, the quantity obtained by reversing the errors of the second limb, adding on the errors of the first limb and dividing by the whole number of observations. It is evident that terms in sin D and sin 2D will contribute to the apparent error of semi-diameter.

I copy here the left-hand sides of the normal equations

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obtained in my December paper, where I attempted to separate the semi-diameter, parallactic inequality, and variation.

$$+5647\mu+3788\delta_1-2327\delta_2\ (=5647\text{ apparent }\mu)$$
$$+3788\mu+3074\delta_1-1366\delta_2\ (=3074\text{ apparent }\delta_1)$$
$$-2327\mu-1366\delta_1+2645\delta_2\ (=2645\text{ apparent }\delta_2)$$

On analysing the apparent coefficient of sin D and the apparent correction to semi-diameter for a period $\omega-\omega'$ I obtain

(in the coefficient of sin D) $+0''{\cdot}29\cos(\omega-\omega')$
(in the correction to semi-diameter) $+0''{\cdot}19\cos(\omega-\omega')$

The first of the normal equations shows that a real inequality $+0''{\cdot}29\cos(\omega-\omega')$ in the coefficient of sin D will produce an apparent inequality $+0''{\cdot}29\times3788\div5647=+0''{\cdot}19\cos(\omega-\omega')$ in the apparent correction to semi-diameter. Hence the conclusion is that there is no periodic inequality in the true correction to semi-diameter, and that the correction $-0''{\cdot}29\cos(\omega-\omega')\sin D$ is really required by the tabular places.

No attempt has yet been made to analyse the observations for a term cos D. On account of the grouping of the observations round full Moon the analysis will clearly be subject to an increased accidental error. Hence it is impossible to say at present in what proportions the term $-0''{\cdot}29\cos(\omega-\omega')\sin D$ must be distributed between arguments $g-g'$ and $g-g'+2\omega-2\omega'$.

The following are the coefficients of sin $(g-g')$ and sin $(g-g'+2\omega-2\omega')$ according to various authorities :

	$\sin(g-g')$.	$\sin(g-g'+2\omega-2\omega')$.
Hansen's tables	$+148''{\cdot}03$	$-28''{\cdot}60$
" theory	$+148{\cdot}02$	$-28{\cdot}56$
Delaunay (1)	$+147{\cdot}46$	$-29{\cdot}50$
" (2)	$+148{\cdot}43$	$-28{\cdot}32$

The last three lines are taken from Newcomb's paper (*Astron. Papers Amer. Eph.* vol. i.). Delaunay (1) and Delaunay (2) are defined in that paper. The difference is the result of estimating for the higher powers not calculated by Delaunay. It is well known that in these terms (that is, terms with characteristic ee') the convergence of Delaunay's series is extremely slow. The present paper shows that in respect of these terms Hansen's theory is considerably nearer the observations than either Delaunay (1) or Delaunay (2).

Lastly I have solved for "true" (in contradistinction to

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"apparent") corrections to the coefficient of sin D and the semi-diameter by the formulæ

$$\mu = 5\cdot77 \text{ apparent } \mu - 3\cdot87 \text{ apparent } \delta_1$$
$$\delta_1 = 5\cdot77 \text{ apparent } \delta_1 - 7\cdot11 \text{ apparent } \mu$$

and the results are given in the last two columns.

From the normal equations above quoted it will be seen that the left-hand sides are strictly $\mu - 0\cdot7\delta_2$ and $\delta_1 + 0\cdot3\delta_2$; but as $\delta_2 = +0''\cdot03$ the difference is unimportant.

Since the December paper was written I have corrected the observations individually for an error of semi-diameter $0''\cdot4$. Judging from that paper, therefore, the solution to be expected is

$$\mu = -0''\cdot054 \qquad \delta_1 = -0''\cdot120$$

From the means of the last two columns in the present paper I get

$$\delta_1 + 0''\cdot01 = -0''\cdot021$$
$$\mu - 0''\cdot02 = -0''\cdot015$$

differing very slightly from the previous values.

Adding the last four columns without regard to sign I obtain for the probable error of the results of the separate periods

for δ_1 (apparent) ... ... $211 \div 48 \times 0\cdot85 = \pm 0''\cdot38$
,, μ ,, ... $148 \div 48 \times 0\cdot85 = \pm 0''\cdot26$
,, δ_1 ... ... $292 \div 48 \times 0\cdot85 = \pm 0''\cdot52$
,, μ ... ... $157 \div 48 \times 0\cdot85 = \pm 0''\cdot28$

From the method of formation of the first of these quantities ($\pm 0''\cdot38$) there is no reason why it should be larger than the mean ($\pm 0''\cdot23$) of the four similarly formed quantities previously obtained. That it is larger is perhaps evidence that the errors are not entirely accidental with reference to the argument D, or that there are time changes from one period of analysis to another, such as might be introduced by personality in semi-diameter and a change of observers. The second quantity is also large, considering that it has double weight. $0\cdot26 \times \sqrt{2} = 0\cdot36$, and this quantity is therefore comparable with the preceding. The third quantity is determined with a weight diminished in the ratio 1030 : 5647 (see December *Monthly Notices*, footnote); $\pm 0''\cdot52$ is in close agreement therefore with $\pm 0''\cdot23$ obtained previously found. The fourth quantity is determined with a weight diminished in the ratio 1562 : 5647; $\pm 0''\cdot28$ is therefore comparable with a probable error $\pm 0''\cdot15$ in the coefficients of periodic inequalities. This additional evidence is therefore against the hypothesis that there are time-changes, for it appears

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that the values of μ are more accordant than might have been expected, having regard to accidental error only.

On analysing the last column but one for argument ☊ I find

(tabular *minus* observed) (+0″·40 sin ☊ − 0″·12 cos ☊) sin D

The former term is far too large to be accidental; but as sin ☊ sin D splits into cosines, and not sines, it is probable that ☊ is not the actual argument of the periodicity, but only a near approximation to it. The point is accordingly held over till the Airy period is reduced. As far as I can foresee, this reduction should be complete in about two months.

There is a point to which attention should be called, although I am at present unable to offer an explanation. The *apparent* coefficient of sin D and the *apparent* correction to semi-diameter are naturally of the same sign as a rule; in fact, they are never of unlike sign. One test of the validity of the solution into true corrections is that there shall be no periodicity in the correction to semi-diameter, but that all the periodicity should belong to the correction to parallactic inequality. The results stand this test. There is, however, another test, viz. that the signs should be independent. Now in the forty-eight periods we have thirty-two cases of unlike sign, six of like sign, and ten cases where one at least of the quantities is zero. This test is therefore not satisfied. As a similarity of sign has been replaced by, on the whole, a dissimilarity, I conclude that the process of separation has been overdone.

Note on Elliptic Motion. By Asaph Hall.

The difference of the ratio of the radius vector and the mean distance from unity and the difference of the true and mean anomalies, or the equation of centre, are two important quantities in the theory of this motion. If these quantities are wanted in explicit terms of the eccentricity and the mean anomaly, perhaps the easiest method is to apply the theorem of Lagrange and to use the equation

$$dv = \sqrt{1-e^2}\cdot\frac{a^2}{r^2}\cdot dnt.$$

The ratio $\frac{a^2}{r^2}$ can be found by the above theorem, and a substitution and integration will give the equation of centre, or $v - nt$. Bessel objected to this method as not being the simple and natural solution of the question.

In his *Mécanique* Poisson has indicated a method of obtaining these quantities by definite integrals, and this is the method